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Determination of Momentum and Angles of Bubble-Chamber Tracks in a Nonuniform Magnetic Field[1]

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ABSTRACT

A method of calculating momentum components for bubble-chamber tracks in a nonuniform magnetic field has been developed. This method has been incorporated into a FORTRAN subroutine (TRED 2) which is now a part of a working version of the spatial reconstruction program TRED. Examples of results are given. Typical computer time is about 0.2 sec per track on IBM 7094.

INTRODUCTION

Magnetic field nonuniformity greatly complicates the geometrical reconstruction of bubble-chamber tracks which for a uniform field are simply helices. There are many approaches to this problem [1]–[4]. In this paper, we present a method to treat rather general nonuniform magnetic fields.

The principle is to find a momentum and angles in such a way that "theoretical" coordinates of the track solved from the equation of motion with these very *same* momentum and the angles give minimum deviation from spatial reconstructed coordinates of measured points. In this paper, we describe this method applying for the geometrical program, TRED. Only a routine (TRED 2) in TRED was modified for this problem.[2]

This routine needs the following steps:

(1) We find space coordinates from measurements and get rough values of momentum and angles at some starting point, say x_0, y_0, x_0. These quantities may be obtained from another routine in the reconstruction program.

[1] Work supported by the U.S. Atomic Energy Commission.

[2] Principally, this method can be applied for other geometrical programs such as HGEOM, THRESH, and FOG by adding or modifying a routine in the programs.

(2) We feed these quantities into this routine. Then, the routine calculates a chi-squared value (χ^2) which is defined below and obtains corrections to the initial values of momentum, angles and coordinates of the starting point by making χ^2 minimum by the variation method. Iterations are made if higher accuracy is necessary.

Letting M be a number of points measured for a track, χ^2 is expressed by

$$\chi^2 = \sum_{i=1}^{M} \left\{ \frac{|X_i - x(s_i)|^2}{\sigma_x^2} + \frac{|Y_i - y(s_i)|^2}{\sigma_y^2} + \frac{|Z_i - z(s_i)|^2}{\sigma_z^2} \right\}; \tag{1}$$

σ_x, σ_y, and σ_z are the assigned errors. X_i, Y_i, and Z_i are the coordinates of the ith measurement point. $x(s_i)$, $y(s_i)$ and $z(s_i)$ are the "theoretical" coordinates at a given track length s_i which are obtained from a solution of the equation of motion with initial values of coordinates, momentum, and direction. In this calculation, we assume that the coordinates X_i, Y_i, and Z_i are independent measured quantities with the errors σ_x, σ_y, and σ_z.[3]

In the program, the initial values of coordinates, x_0, y_0, and z_0, are taken as the first measurement point. We define two vectors $\mathbf{R}_i$ and $\mathbf{r}(s_i)$ as

$$\mathbf{R}_i = (X_i, Y_i, Z_i),$$
$$\mathbf{r}(s_i) = (x(s_i), y(s_i), z(s_i)).$$

x_0, y_0 and z_0 are given in vector notation:

$$\mathbf{R}_1 = (x_0, y_0, z_0) = \mathbf{r}(s_1).$$

The track length s_i is approximated as the sum of track segments from the first point to the ith point taking the segments to be straight lines joining adjacent points,

$$s_i = \sum_{j=1}^{i} |\mathbf{R}_j - \mathbf{R}_{j-1}|, \tag{2}$$

where we set $\mathbf{R}_1 = \mathbf{R}_0$.

Because of this approximation of s_i, the deviation $|\Delta\mathbf{r}_i|$, which is the shortest

[3] For the recent programs such as HGEOM and TVGP, the χ^2 should be expressed on film planes taking deviations of measured coordinates from the particle orbit $\mathbf{r}(s)$. Since our calculation was incorporated into the program, TRED, which gives space coordinates (X_i, Y_i, Z_i), we have used the expression (1) as a χ^2. The errors σ_x, σ_y and σ_z are assumed to be constant as

$$\sigma_x = \sigma_y = \sigma_z / L.$$

The factor L can be somewhat chamber-dependent and is taken to be $L = 3$ as being suitable for the Brookhaven 80-in. bubble chamber.

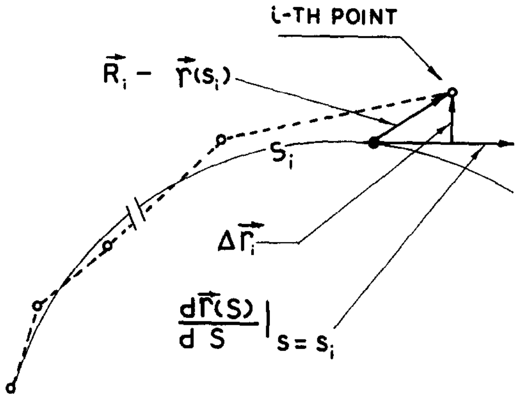

FIG. 1. Particle trajectory and measured points showing the first-order correction to $\Delta \mathbf{r}_i$.

distance from the ith point to the "theoretical" particle trajectory should be corrected to the first order of s. In vector notation (see Fig. 1),

$$\Delta \mathbf{r}_i = \mathbf{R}_i - \mathbf{r}(s_i) - \left[(\mathbf{R}_i - \mathbf{r}(s_i)) \cdot \left(\frac{d\mathbf{r}(s)}{ds}\right)_{s=s_i}\right] \cdot \left(\frac{d\mathbf{r}(s)}{ds}\right)_{s=s_i}. \tag{3}$$

This quantity replaces $(\mathbf{R}_i - \mathbf{r}(s_i))$ in Eq. (1).

We can expand $\mathbf{r}(s)$ to the first order in terms of the five track parameters $(p_0, \lambda_0, \varphi_0, y_0, z_0)$[4]:

$$\begin{aligned} \mathbf{r}(s_i) = \mathbf{r}(s)| \, & p_0, \lambda_0, \varphi_0, y_0, z_0 \\ & + \frac{\partial \mathbf{r}}{\partial p_0} \cdot \delta p_0 + \frac{\partial \mathbf{r}}{\partial \lambda_0} \cdot \delta \lambda_0 + \frac{\partial \mathbf{r}}{\partial \varphi_0} \cdot \delta \varphi_0 \\ & + \frac{\partial \mathbf{r}}{\partial y_0} \cdot \delta y_0 + \frac{\partial \mathbf{r}}{\partial z_0} \cdot \delta z_0 , \end{aligned} \tag{4}$$

where y_0 and z_0 are the coordinates of the first point and p_0, λ_0, and φ_0 are the momentum, the dip angle and the azimuth angle of the track at this point, respectively.

[4] This choice of the parameters $(p_0, \lambda_0, \varphi_0, y_0, z_0)$ is suitable when the track is in forward direction (x-axis). In the least-squares fit, five parameters are sufficient to vary independently. If we use six parameters $(p_0, \lambda_0, \varphi_0, x_0, y_0, z_0)$, the determinant (6×6) for this calculation is zero. The author would like to thank Dr. Arthur H. Rosenfeld and Dr. Frank T. Solmitz for pointing out this problem.

Introducing the notation:

$$t_1 = p_0, \quad t_2 = \lambda_0, \quad t_3 = \varphi_0, \quad t_4 = y_0, \quad t_5 = z_0,$$

the conditions that χ^2 is minimum are given by

$$\partial\chi^2/\partial(\delta t_i) = 0 \qquad (i = 1, 2, \ldots, 5) \tag{5}$$

Thus, we will have the following five simultanious equations from (3), (4), and (5):

$$\sum_{i=1}^{5} D_{ij}\,\delta t_j = D_i \qquad (i = 1, 2, \ldots, 5), \tag{6}$$

where

$$D_{ij} = \sum_{k=1}^{M} \frac{\partial \mathbf{V}_k}{\partial t_i} \cdot \frac{\partial \mathbf{V}_k}{\partial t_j},$$

$$D_i = \sum_{k=1}^{M} \frac{\partial \mathbf{V}_k}{\partial t_i} \cdot \Delta \mathbf{V}_k .$$

Here,

$$(V_k)_x = \frac{X_k - \Delta x_k}{\sigma_x}, \qquad (V_k)_y = \frac{Y_k - \Delta y_k}{\sigma_y}, \qquad (V_k)_z = \frac{Z_k - \Delta z_k}{\sigma_z},$$

$$(\Delta V_k)_x = \frac{\Delta x_k}{\sigma_x}, \qquad (\Delta V_k)_y = \frac{\Delta y_k}{\sigma_y}, \qquad (\Delta V_k)_z = \frac{\Delta z_k}{\sigma_z},$$

where $(\Delta x_k, \Delta y_k, \Delta z_k) = \Delta \mathbf{r}_k$ in Eq. (3).

Solving these equations, the corrections to the initial values of $y_0, z_0, p_0, \lambda_0, \varphi_0$, will be obtained, namely,

$$\delta t_i = \Delta_i/\Delta \qquad (i = 1, 2, \ldots, 5), \tag{7}$$

where Δ is the determinant of the array D_{ij} and Δ_i is the determinant of the array D_{ij} where the jth column is replaced by D_i:

$$\Delta_j = \begin{vmatrix} D_{11} & D_{12} & \cdots & D_1 & \cdots & D_{15} \\ D_{21} & D_{22} & & D_2 & & D_{25} \\ D_{31} & D_{32} & & D_3 & & D_{35} \\ D_{41} & D_{42} & & D_4 & & D_{45} \\ D_{51} & D_{52} & & D_5 & & D_{55} \end{vmatrix}. \tag{8}$$

The expression of $\mathbf{r}(s)$ and derivatives of $\mathbf{r}(s)$ in Eq. (4) will be given in the next section.

1. The Particle Tracjectory in an Arbitrary Magnetic Field

In order to carry out the procedure outlined in the preceeding section, we use power-series expansions for the variables. The functional form of $\mathbf{r}(s)$ can be obtained from the relativistic equation of motion of a particle of mass m in the nonuniform magnetic field $\mathbf{B}$:

$$\frac{d\mathbf{p}}{dt} = \frac{e}{c} \cdot [\mathbf{v} \times \mathbf{B}] - f(s) \cdot \mathbf{v}, \tag{9}$$

where $f(s) \cdot \mathbf{v}$ is the momentum loss per second due to ionization. The radiation loss due to deceleration of a charged particle is neglected in Eq. (9) (the loss is of order 0.001 MeV per second for a 2BeV/c π-meson in a magnetic field of 20 kG.)

Using the relations

$$\begin{aligned} \mathbf{p}(s) &= p(s) \cdot (\mathbf{v}/v) = p(s) \cdot \mathbf{n}(s), \\ \mathbf{n}(s) &= (\mathbf{v}/v) = d\mathbf{r}(s)/ds, \\ d\mathbf{p}/dt &= v[p(s) \cdot d\mathbf{n}/ds + \mathbf{n}(s) \cdot dp(s)/ds], \end{aligned}$$

we obtain two equations from Eq. (9),

$$dp(s)/ds = -f(s), \tag{10}$$

$$p(s) \cdot d\mathbf{n}(s)/ds = (e/c) \cdot [\mathbf{n}(s) \times \mathbf{B}(s)]. \tag{11}$$

Using the well known range–momentum relation, we can express $p(s)$ as a power series in s:

$$p(s) = p_0 + p_1 s + p_2 s^2 + \cdots, \tag{12}$$

where the coefficients p_0, p_1, p_2 are determined for each track by the range–momentum relation. It is noted that p_1 and p_2 are dependent on particle mass. Similarly, we can express $\mathbf{B}(s)$ as a power series in s:

$$\mathbf{B}(s) = \mathbf{B}_0 + \mathbf{B}_1 \cdot s + \mathbf{B}_2 \cdot s^2 + \cdots, \tag{13}$$

where $\mathbf{B}_0$, $\mathbf{B}_1$, $\mathbf{B}_2$ ... can be determined by fitting $\mathbf{B}(s)$ to measured values of the magnetic field for each track.

The solution of Eq. (11) is now given as a power series in s:

$$\mathbf{n}(s) = d\mathbf{r}(s)/ds,$$

$$\mathbf{r}(s) = \sum_{n=0}^{\infty} \mathbf{a}_n \cdot s^n \tag{14}$$

where the coefficients $\mathbf{a}_n$ are given in terms of the $\mathbf{B}_n$ and p_n:

$$\mathbf{a}_0 = (x_0, y_0, z_0) \qquad \text{(initial position)}$$

$$\mathbf{a}_1 = (\cos\lambda_0 \cos\varphi_0, \cos\lambda_0 \sin\varphi_0, \sin\lambda_0) \qquad \text{(initial direction)}$$

$$\mathbf{a}_2 = \frac{1}{2p_0}\frac{e}{c} \cdot [\mathbf{a}_1 \times \mathbf{B}_0]$$

$$\mathbf{a}_3 = \frac{1}{6p_0} \cdot \left[-2\mathbf{a}_2 p_1 + \frac{e}{c}(\mathbf{a}_1 \times \mathbf{B}_1 + 2\mathbf{a}_2 \times \mathbf{B}_0)\right]$$

$$\cdots$$

$$\mathbf{a}_n = \frac{1}{n(n-1)\,p_0}\left\{-\sum_{i=1}^{n-2}(n-i)(n-1-i)\,\mathbf{a}_{n-i}p_i + \frac{e}{c}\sum_{i=1}^{n-1}(n-i)(\mathbf{a}_{n-i} \times \mathbf{B}_{i-1})\right\} \tag{15}$$

It is noted that

$$|\mathbf{n}(s)|^2 = 1.$$

The derivatives of $\mathbf{r}(s)$ with respect to p_0, λ_0, φ_0, y_0, z_0 are given using the notation t_i:

$$\frac{\partial \mathbf{r}(s)}{\partial t_i} = \sum_{n=0}^{\infty} \frac{\partial \mathbf{a}_n}{\partial t_i} \cdot s^n. \tag{16}$$

Examples of derivatives, $\partial\mathbf{a}_n/\partial p_0$, $\partial\mathbf{a}_n/\partial\lambda_0 \cdots$ are shown below:

$$\frac{\partial \mathbf{a}_0}{\partial p_0} = 0, \qquad \frac{\partial \mathbf{a}_1}{\partial p_0} = 0, \qquad \frac{\partial \mathbf{a}_2}{\partial p_0} = -\frac{\mathbf{a}_2}{p_0},$$

$$\frac{\partial \mathbf{a}_0}{\partial \lambda_0} = 0, \qquad \frac{\partial \mathbf{a}_1}{\partial \lambda_0} = (-\sin\lambda_0 \cos\varphi_0, -\sin\lambda_0 \sin\varphi_0, \cos\lambda_0)$$

$$\frac{\partial \mathbf{a}_0}{\partial \varphi_0} = 0, \qquad \frac{\partial \mathbf{a}_1}{\partial \varphi_0} = (-\cos\lambda_0 \sin\varphi_0, \cos\lambda_0 \cos\varphi_0, 0)$$

$$\frac{\partial \mathbf{a}_0}{\partial y_0} = (0, 1, 0), \qquad \frac{\partial \mathbf{a}_n}{\partial y_0} = \frac{\partial \mathbf{a}_n}{\partial z_0} = 0 (n \geqslant 1),$$

$$\frac{\partial \mathbf{a}_0}{\partial z_0} = (0, 0, 1),$$

2. Error Estimation and Discussion

For practical calculation we approximate $\mathbf{r}(s)$ by a finite number of terms. This approximated $\mathbf{r}(s)$ causes a deviation from the true $\mathbf{r}(s)$ which gives errors to the fitted momentum and angles.

In order to show relations between the fitted momentum error and the number of terms, N, used for the approximated $\mathbf{r}(s)$, we will give an expression of $\mathbf{r}(s)$ in a very simple case, namely, a constant magnetic field ($Bx = By = 0$, $Bz = B$). The starting point (x_0, y_0, z_0) and the initial direction are taken as $x_0 = z_0 = 0$, $y_0 = \rho$ and

$$\left(\frac{dy}{ds}\right)_{s=0} = \left(\frac{dz}{ds}\right)_{s=0} = 0, \qquad \left(\frac{dx}{ds}\right)_{s=0} = 1,$$

Thus, $\mathbf{B}(s) = (0, 0, B)$, $\mathbf{a}_0 = (0, \rho, 0)$, $\mathbf{a}_1 = (1, 0, 0)$, where ρ is the radius of curvature ($p = (e/c)\,B\rho$) and the momentum loss due to ionization is neglected. (see Fig. 2).

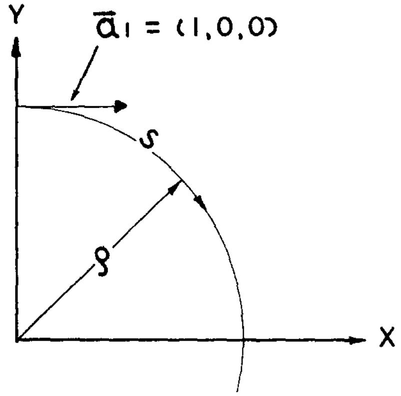

Fig. 2. Particle trajectory in a constant magnetic field without the momentum loss due to ionization.

We can then calculate $\mathbf{a}_n$ for all n. We find

$$\mathbf{r}(s) = \begin{pmatrix} x(s) \\ y(s) \\ z(s) \end{pmatrix} = \begin{pmatrix} \rho\left[\frac{s}{\rho} - \frac{1}{3!}\left(\frac{s}{\rho}\right)^3 + \frac{1}{5!}\left(\frac{s}{\rho}\right)^5 - \cdots\right] \\ \rho\left[1 - \frac{1}{2!}\left(\frac{s}{\rho}\right)^2 + \frac{1}{4!}\left(\frac{s}{\rho}\right)^4 - \cdots\right] \\ 0 \end{pmatrix}.$$

The factorial coefficients in Eq. (17), make $\mathbf{r}(s)$ a very strongly convergent function of s. Therefore, if we approximate $\mathbf{r}(s)$ by taking only the first N terms, the deviation from the true $\mathbf{r}(s)$ will be approximated by the $(N+1)$th term in Eq. (17), namely,

$$\Delta r \approx \rho(s/\rho)^{(N+1)}/(N+1)!\,.$$

Using the relation that sagitta $= s^2/8\rho$, the fractional error in fitted momentum at a distance s along the track is of order of magnitude

$$(\Delta p/p)_N = 8(s/\rho)^{N-1}/(N+1)!\,. \tag{18}$$

In the case of a nonuniform field, we may approximate the deviations of $p(s)$ and $\mathbf{B}(s)$ from the true $p(s)$ and $\mathbf{B}(s)$ by the $(N'+1)$th and $(N''+1)$th term in Eq. (12) and (13), respectively. Letting $\Delta P_{N'}$ and $\Delta B_{N''}$ be these errors in fitted momentum due to the finite power-series expansions, the fractional errors of $p(s)$ and $B(s)$ are given by

$$(\Delta p/p)_{N'} = 8(\Delta p_{N'}/p)/(N'+3)(N'+2), \tag{19}$$

$$(\Delta p/p)_{N''} = 8(\Delta B_{N''}/B)/(N''+3)(N''+2), \tag{20}$$

where N' and N'' are the numbers of terms used for the approximated $p(s)$ and $\mathbf{B}(s)$. Similar consideration can be applied to the errors of angles due to the approximations of $p(s)$, $\mathbf{B}(s)$ and $\mathbf{r}(s)$.

The choice of the numbers N, N', and N'' depends upon the accuracy required in an experiment. These numbers should be chosen in such a way that the errors of fitted momentum and angles due to the approximations of $p(s)$, $\mathbf{B}(s)$, and $\mathbf{r}(s)$ are less than other errors—for instance, less than measurement error. For an 8 BeV/c π^-p experiment which was performed in the B.N.L. 80-in. bubble chamber, the $p(s)$, $\mathbf{B}(s)$, and $\mathbf{r}(s)$ were chosen as follows:

$$\begin{aligned} p(s) &= p_0 + p_1 + p_1 s + p_2 s^2, \\ \mathbf{B}(s) &= \mathbf{B}_0 + \mathbf{B}_1 s + \mathbf{B}_2 s^2 + \mathbf{B}_3 s^3, \\ \mathbf{r}(s) &= \mathbf{a}_0 + \mathbf{a}_1 s + \cdots + \mathbf{a}_7 s^7. \end{aligned} \tag{21}$$

Due to these approximations the error $\Delta p/p$ is of order 0.1%, and the errors of angles are of order 0.002°, for a pion momentum 100 MeV/c and greater, if the track length is less than the radius curvature. Fitted quantities (p, λ, φ) for a generated track are listed in the Appendix. The results using a real track are also given.

As can be seen from Eq. (17), the $\mathbf{r}(s)$ is convergent even if s/ρ is larger than one. However, it is desirable to use a track length not larger than the radius of curvature, since the error $\Delta p/p$ is proportional to $(s/\rho)^{N-1}$. If we take the point (x_0, y_0, z_0) at the midpoint of the track, we can extend the track length from $-s$ to $+s$, so that the track length can be extended to twice ρ keeping $s/\rho \leqslant 1$. In any event, if the track curves as much as 1 radian, the momentum error, in existing bubble chambers, is likely to be dominated by Coulomb scattering—so a track length greater than ρ is not particularly helpful.

The computer time to calculate the momentum and angles depends on how many terms are used for $p(s)$, $\mathbf{B}(s)$ and $\mathbf{r}(s)$, and how many points are measured for the track. The time also depends on the number of iterations and the number of particle masses used for the calculation. Using Eq. (21) for $p(s)$, $\mathbf{B}(s)$ and $\mathbf{r}(s)$, the IBM 7094 computer time for one iterations and one particle mass is about 0.2 sec per 8-point track.

Acknowledgments

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Appendix

We will show two examples of how this program makes corrections to the starting values p_0, λ_0, φ_0, and z_0.[5]

As the first example, we use a generated track of a 100 MeV/c π meson, where we assume that 8 points are measured along the track in equal spacing and that the track length is 14.0 cm. The coordinates of the 8 points are listed in the first part of Table I with values of a magnetic field at each point which is assumed to be the magnetic field of the B.N.L. 80-in. bubble chamber [5]. No measurement and multiple scattering errors are taken into account for this generated track. We then feed these coordinates and crude starting values of momentum and angles into this fitting program, where the starting values are taken as

$$p_0 = 112.54 \text{ MeV}/c, \qquad \lambda_0 = 79.18°,$$
$$\varphi_0 = 30.29°, \qquad z_0 = 35.00 \text{ cm}.$$

Results of the calculation at each iteration are listed in the second part of Table I. It is noted that the fitted momentum converges rapidly to the true

[5] We have used corrections to only four track parameters p_0, λ_0, φ_0, and z_0 to reduce the programming complication in our program. The correction Δz_0 is correlated with $\Delta\lambda_0$, whereas $\Delta\lambda_0$ is correlated with Δp_0 and $\Delta\varphi_0$. This approximation, therefore, gives better correction to λ_0 than to p_0 and φ_0.

TABLE I

COORDINATES AND MAGNETIC FIELD AT EIGHT POINTS ALONG A GENERATED TRACK (100 MeV/c π^-)

P(MeV/c)	X(cm)	Y(cm)	Z(cm)	B_x(kG)	B_y(kG)	B_z(kG)
100.00	−30.000	0.	35.000	−0.379	−0.121	20.365
98.02	−29.808	0.287	36.970	−0.376	−0.125	20.445
95.96	−29.580	0.543	38.940	−0.372	−0.131	20.527
93.81	−29.320	0.763	40.911	−0.369	−0.139	20.611
91.57	−29.032	0.943	42.882	−0.366	−0.148	20.696
89.20	−28.720	1.077	44.853	−0.362	−0.160	20.782
86.71	−28.391	1.161	46.824	−0.360	−0.175	20.870
84.05	−28.052	1.192	48.794	−0.357	−0.194	20.959

RESULTS

	P_0(MeV/c)	λ_0(degrees)	φ_0(degrees)	z_0(cm)
True Values	100.0	80.00	30.00	35.000
Starting Values	112.54	79.18	30.29	35.000
1st iteration	97.53	80.06	30.19	34.996
2nd iteration	100.41	80.01	29.96	34.996
3rd iteration	99.98	80.00	30.01	34.996
4th iteration	100.05	80.00	30.00	34.996

momentum and the ratio of deviations, $(p_{\text{fitted}} - p_{\text{true}})^{(i+1)}/(p_{\text{fitted}} - p_{\text{true}})^{(i)}$, for successive iterations is about 1 : 5 up to the second iteration. The deviation of z_0 from the true value of z_0 after the second iteration is 0.004 cm and this comes from the fact that the track used in this example has a large dip angle.

The determination of the useful number of iterations used for this calculation depends on the crudeness of the starting values and the errors of the xyz-coordinates of data points due to measurement and multiple scattering. The second example which uses a real track illustrates this point.

Using an 8-BeV/c π^-p picture taken at the B.N.L. 80 in. bubble chamber, we measured 7 points along a π^+ track on our digitizing machine (HERMES). Coordinates of these points are converted into the spatial coordinates through the B.N.L. spatial reconstruction program, TRED [6] (see first part of Table II). TRED also calculates momentum and angles at the beginning point of the track by using only the z-component of magnetic field ($Bx = By = 0$). These calculated momentum and angle values are used as the starting values for the fitting program TRED 2. Results are listed in the second part of Table II with errors of momentum and angles due to measurement and multiple scattering [7].

TABLE II

RECONSTRUCTED COORDINATES OF MEASURED POINTS AND MAGNETIC FIELD ALONG AN ACTUAL π^+ TRACK

Track Length (cm)	X(cm)	Y(cm)	Z(cm)	B_x(kG)	B_y(kG)	B_z(kG)
0.00	−62.4117	2.3982	29.6463	.701	−.136	20.422
1.6239	−62.3508	1.1753	30.7130	.701	−.109	20.454
6.1775	−61.9771	−2.2694	33.6677	.698	−.026	20.543
15.7491	−60.2828	−9.3829	39.8436	.681	.194	20.725
27.6957	−56.4572	−17.6685	47.5531	.631	.568	20.948
32.4244	−54.3961	−20.6878	50.5524	.600	.741	21.038
43.0668	−48.7743	−26.7734	57.2315	.509	1.178	21.272

RESULTS

	P_0(MeV/c)	λ_0(degrees)	φ_0(degrees)	z_0(cm)
Starting values	335.25	39.88	−88.44	29.6463
1st iteration	336.80	40.68	−88.14	29.6324
2nd iteration	337.72	40.67	−88.08	29.6378
Error	4.0	0.21	0.10	

As seen in Table II, second part, the corrections to momentum and angles at the second iteration are less than the errors, so that it is not necessary to make further iterations. It is also noted that neglect of the small componentsof magnetic field, B_x and B_y, causes the same order of error of momentum as that of measurement and multiple scattering in this case.

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